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Gamma Rays and the Decay of Neutrinos from SN1987A

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Abstract

We calculate limits to the properties of massive, unstable neutrinos using data from gamma-ray detectors on the Pioneer Venus Orbiter Satellite; a massive neutrino emitted from SN1987A that decayed in flight and produced gamma rays would be detectable by this instruments. The lack of such a signal allows us to constrain the branching ratio to photons (B_γ), mass (m_ν), and radiative lifetime ($\tau_\gamma = \tau/B_\gamma$). For low mass ($m < T \sim 8$ MeV) neutrinos decaying $\nu \rightarrow \nu'\gamma$, $B_\gamma < 3 \times 10^{-7}$, for $m_\nu\tau \lesssim 10^6$ keV sec, and $B_\gamma < 6 \times 10^{-14}m_\nu\tau$ / keV sec for $m_\nu\tau \gtrsim 10^6$ keV sec; limits for high-mass neutrinos are somewhat weaker due to Boltzmann suppression. We also calculate limits for decays that produce gamma rays through the bremsstrahlung channel, $\nu \rightarrow \nu'e^+e^-\gamma$. In the case that neutrino mass states are nearly degenerate, $\delta m^2/m^2 \ll 1$, our limits for the mode $\nu \rightarrow \nu'\gamma$ become more stringent by a factor of $\delta m^2/m^2$, because more of the decay photons are shifted into the PVO detector energy window.

I. INTRODUCTION

The occurrence of Supernova 1987A in the Large Magellanic Cloud has proven to be among the most fruitful experiments in the heavenly laboratory for confirming "known" physics and constraining new physics. Aside from its obvious impact upon the study of the late stages of stellar evolution in general and upon supernova physics in particular, models for SN1987A have become a test bed for the study of the couplings of light particles (neutrinos, axions) to ordinary matter [1]. In this work, we discuss limits on the properties



of neutrinos independent of a specific model for the supernova based upon the emission of thermal neutrinos from the hot nascent neutron star.

When a supernova occurs, the bulk of the binding energy of the progenitor star ($\sim 3 \times 10^{53}$ erg) is released in neutrinos, as predicted by theory and confirmed by the observation of a neutrino burst from SN1987A, with a characteristic temperature of about $T_\nu \approx 4.5$ MeV (for low-mass electron neutrinos; other species are predicted to have a higher temperature, $T \simeq 8$ MeV, because of their different coupling to the prevalent electrons [7,8]). If at least one species of neutrinos is unstable and couples to the photon, then some of these neutrinos will decay to photons en route, which are potentially detectable as MeV gamma rays. At the time of the supernova burst's arrival at earth and environs, there were several satellites operating in the solar system capable of detecting the decay photons in the course of their watch for gamma-ray bursts. Analyses of the data from one of these detectors, on board the Solar Max Mission (SMM) satellite, has already been presented [2]. Here, we examine the data from the Gamma Burst Detector on the Pioneer Venus Orbiter (PVO). While the PVO detector was smaller, and its energy window is not well-matched to that of the supernova neutrinos, it had 4π acceptance and was in an environment free of the Earth's radiation belts (leading to lower backgrounds). In addition, more high quality data is available (> 8000 sec vs. 10 sec for SMM).

The paper is organized as follows. In the next section an exact formula for the expected gamma-ray flux is derived and important approximations are developed. In Section III, the PVO data are discussed and rigorous limits are derived in the simplest regime. The next four sections build upon these results, expanding to more complicated regimes. The final section is a brief summary.

II. GAMMA-RAY SIGNAL

We can write the expected fluence of gamma rays from decaying neutrinos with mass m_ν and mean lifetime τ as [3]

$$dN = \frac{B_\gamma L_\#(E)}{4\pi D^2} dE n(\mu) d\mu \frac{e^{-t_d/\gamma\tau}}{\gamma\tau} dt_d \times \delta\left(t - t_d \left[1 - v\mu + \frac{D}{t_d} \left\{\sqrt{1 - (vt_d/D)^2 (1 - \mu^2)} - 1\right\}\right]\right) dt. \quad (1)$$

where B_γ is the fraction of decays that produce a gamma ray. The first factor is the overall flux of neutrinos from a supernova at a distance D . $L_\#(E)$ is the differential number flux of neutrinos of energy E , so $E_T = \int dE E L_\#$ is the total luminosity in neutrinos. The second factor gives the fraction that decay into a "lab-frame" angle $\arccos \mu$. The third factor gives the fraction that decay at time t_d . Finally, the delta function selects the photons with a given t_d , E , μ that arrive at a time t after the arrival of massless neutrinos at the detector. The Lorentz factor is $\gamma = E/m_\nu$ and the speed $v = \sqrt{1 - \gamma^{-2}}$.

The function $n(\mu)$ depends on the distribution of daughter photons in the neutrino rest frame, and therefore on the particular decay channel involved. First, we consider the two-body decay, $\nu \rightarrow \nu' \gamma$. Because the neutrino is a spin-1/2 particle, and the photon a spin-1 particle, this reaction can proceed in one of two ways: with the helicity of the daughter

neutrino either parallel or antiparallel the initial helicity. From quantum mechanics, then, the distribution of the photon in the rest frame of the parent will be proportional to either $(1 \pm \bar{\mu})/2$, where $\bar{\mu}$ is the cosine of the rest-frame angle between the directions of the parent neutrino and the photon. Transforming into the lab frame gives us the distribution $n(\mu)$. Note that we have assumed the neutrinos are ejected from the supernova in an instantaneous burst; as long as the actual duration of the pulse is small compared to the timing resolution of the detector, which is the case, this is a good approximation.

Because we are not interested in the decay angle, but rather the photon energy, we write $n(\mu)d\mu = f(E, k)dk$, where k is the gamma-ray energy, related to the decay angle by

$$\mu = \frac{\gamma}{\sqrt{\gamma^2 - 1}} \left(1 - \frac{m}{2\gamma k} \right). \quad (2)$$

This gives

$$f(E, k) = \frac{1}{(Ev)^2} (Ev \mp E \pm 2k), \quad (3)$$

for each of the helicity possibilities. (For reference, an isotropic decay would give $f(E, k) = 1/(Ev) = 1/p$, where p is the neutrino momentum.) For low-mass neutrinos with $v \approx 1$,

$$f(E, k) = \begin{cases} 2k/E^2 & \text{no flip} \\ 2(E - k)/E^2 & \text{flip} \\ 1/E & \text{isotropic} \end{cases}. \quad (4)$$

Each of these should be multiplied by a Heaviside function $\Theta(E - k)$ to require that the daughter photon be less energetic than the parent. Further, we require that the decay does not occur inside of the progenitor envelope which would considerably alter the energetics of the explosion and lead to an independent constraint which is important for short lifetimes [9].

The factor inside the delta function is especially complicated. This occurs because at any given time, the detector is receiving photons from neutrinos that have decayed in a complicated shape, approximately an ellipsoid with the supernova at one focus and the detector at another, further complicated by the speed $v < 1$ of the massive neutrinos. This includes photons that have left the supernova pointing far away from the detector but have decayed at large angles toward the detector. Obviously, for low mass neutrinos which leave the supernova at highly relativistic speeds, the fraction which take such a path is very small. To simplify this expression, we shall require that $t_d \ll D \sim 5 \times 10^{12}$ sec—most of the neutrinos decay well before they reach the earth. Otherwise, the flux is greatly reduced and the limits are correspondingly weaker; the neutrinos may typically take a path that causes them to decay well after our observations end. In Sec. VII we discuss long lifetimes.

In this limit, the delta function becomes simply $\delta(t - t_d(1 - v\mu)) = (2\gamma k/m)\delta(t_d - 2\gamma kt/m)$, and we can perform the integral over t_d :

$$dN = \frac{B_\gamma L_\#(E)}{4\pi D^2} f(E, k) \frac{2k}{m_\nu \tau} e^{-2kt/m_\nu \tau} dt dE dk \quad (5)$$

(Similar expressions have also been derived in [4,5].) We shall assume that the neutrino-number luminosity is given by a zero-chemical-potential Fermi-Dirac distribution with known temperature and total neutrino energy, a reasonable approximation [8]. For now, we will present results for low-mass neutrinos (*i.e.*, $m_\nu \ll T_\nu$),

$$L_\#(E, m_\nu = 0) = \frac{120}{7\pi^4} \frac{E_T}{T_\nu^4} \frac{E^2}{1 + e^{E/T_\nu}}, \quad (6)$$

where $E_T \simeq 10^{53}$ erg is the total energy in one species of massless neutrinos. We treat the case $m_\nu \gtrsim T_\nu$ in Sec. V below.

Finally, we can integrate the above expression over all neutrino energies E and over one time bin from t to $t + \delta t$ to get an expression for the spectrum of photons incident on the detector during that time interval. For $m_\nu \ll T_\nu$,

$$\phi(k, t) = \int_t^{t+\delta t} \frac{dN}{dk dt} dt = \frac{B_\gamma}{4\pi D^2} \frac{240}{7\pi^4} \frac{E_T}{T_\nu^2} h(k/T_\nu) e^{-2kt/m_\nu\tau} (1 - e^{-2k\delta t/m_\nu\tau}). \quad (7)$$

In this expression, the function $h(k/T_\nu)$ results from the integral over the neutrino energies. It is of order unity for the parameter ranges of interest, and it is largest in the case of “no flip,” which we will assume from now on since it gives the most conservative estimates of the parameters. In that case, it is given by $h(y) = y \ln(1 + e^{-y})$.

Although this signal depends nonlinearly on the parameter $m_\nu\tau$, the expression simplifies when $m_\nu\tau$ is much greater or less than kt or $k\delta t$, where k is a typical photon energy, giving

$$\phi(k, t) = \frac{B_\gamma}{4\pi D^2} \frac{240}{7\pi^4} \frac{E_T}{T_\nu^2} h(k/T_\nu) \times \begin{cases} \delta(t), & m_\nu\tau \ll k\delta t \\ 2k\delta t/m_\nu\tau, & m_\nu\tau \gg kt \end{cases}. \quad (8)$$

In the former case of small $m_\nu\tau$, there is no appreciable relativistic delay before the decay of the neutrinos, so essentially all of the daughter photons arrive in the first time bin after the supernova. In the case of large $m_\nu\tau$, the flux is essentially constant over the time of the observations, so the signal is proportional to the width of the time bin. Note that only in the latter case does the fluence actually depend on the value of $m_\nu\tau$.

In order to calculate the expected signal from the theoretical spectrum, we must fold that spectrum with the appropriate response function. The signal expected in the i th energy channel is

$$S_i(t) = \int dk R_i(k) \phi(k, t) = \sum_j R_{ij} \phi_j(t) \quad (9)$$

where R_{ij} is the response of detector i in energy bin k_j , and $\phi_j(t)$ is the theoretical spectrum averaged over energy bin j at time (or time bin) t . This, combined with a separate observation of the background rate in each of the four detectors, provides the theoretical signal to be compared with the observations.

III. GAMMA-RAY DATA AND ANALYSIS

To obtain our limits we use data from the supernova with data from Pioneer Venus Orbiter Gamma Burst Detector (PVO GBD) [6], luckily in operation in February, 1987.

The GBD has four energy channels, roughly 100 – 200 keV, 200 – 400 keV, 500 – 1000 keV, and 1 – 3 MeV towards the direction of the supernova, which was propitiously directly overhead at the time, giving the maximum effective area. We have data for about 1500 sec prior to the supernova lights arrival at Venus (for calculating the background), and for 8000 sec after, for time bins of either 12 or 16 sec in duration. We show the data in Figure 1. We have verified that there is no clear signal in any of the four channels; further, the data is consistent with a constant Poisson rate in each detector.

To calculate limits on our parameters B_γ and $m_\nu\tau$, we use the folded spectrum $S_i(t)$ and our measurement of the background rate (observed for a length of time t_b) in each detector to construct a likelihood function given the observed gamma-ray counts in each detector. We assume that the folded spectrum $S_i(t)$ gives the mean of a Poisson process governing the detected number of counts, and that the rates for each detector are high enough to approximate this by an appropriate Normal distribution, for ease of calculation (in the 1 – 3 MeV bin, with the lowest fluence, there are approximately 40 counts per bin). This gives

$$\mathcal{L}(\theta) = \prod N(D_{ij}; b_i\delta t_j + S_{ij}(\theta), \sigma_{ij}^2) \quad (10)$$

where

$$N(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[-\frac{1}{2} \frac{(x - \mu)^2}{\sigma^2} \right] \quad (11)$$

gives the Normal distribution, b_i is the background rate in detector i (observed for a time t_b), δt_j is the length of time bin j , and D_{ij} , S_{ij} are, respectively, the observed and theoretical signal in those time bins, where the latter is calculated with the set of parameters represented by θ . Finally, the variance is given by $\sigma_{ij}^2 = S_{ij} + b_i\delta t_j(1 + \delta t_j/t_b)$, the sum of the theoretical variance of the signal and that due to the background rate (including a small contribution reflecting the uncertainty in that rate; this latter effect is somewhat more difficult to include if a Poisson distribution is explicitly used but is in any case negligible). We define a χ^2 statistic,

$$\chi^2 \equiv -2 \ln \mathcal{L} + \text{const} = \sum_{ij} \left[\ln \sigma_{ij}^2 + \frac{S_{ij}^2}{\sigma_{ij}^2} + 2 \frac{(b_i\delta t_j - D_{ij}) S_{ij}}{\sigma_{ij}^2} + \frac{(D_{ij} - b_i\delta t_j)^2}{\sigma_{ij}^2} \right] \quad (12)$$

Note that the model is nonlinear, and the variance σ_{ij}^2 depends on the model parameters, so we have defined this quantity including the $\ln \sigma_{ij}$ term and that the usual χ^2 distribution does not apply; instead we must apply Bayes' theorem and integrate over the likelihood to determine confidence intervals on our parameters.

Because of the two terms contributing to the variance, the form of χ^2 depends on which dominates. For $S_{ij} \gg b_i\delta t_j$, $\sigma_{ij}^2 \approx S_{ij}$, and the S_{ij}^2/σ_{ij}^2 term dominates, so $\chi^2 \sim \sum S_{ij}$. When the neutrino signal is small, the background contribution dominates, and $\chi^2 \simeq \text{const}$. These regimes are shown in Figure 2, where we plot χ^2 as a function of $m_\nu\tau$ for several values of B_γ . We have used a neutrino temperature of 8 MeV, appropriate for mu and tau neutrinos.

Immediately, we see the character of the limits on the parameters. For $m_\nu\tau \lesssim 10^7$ keV sec, $\chi^2 \propto B_\gamma$; in this regime only the data from the first time bin after the supernova contributes.

Then $\chi^2 \propto B_\gamma/m_\nu\tau$ for $m_\nu\tau/B_\gamma \lesssim 10^{13}$ keV sec; now, the full data set provides information. Finally, $\chi^2 = \text{const}$ for $m_\nu\tau/B_\gamma \gtrsim 10^{13}$ keV sec; in this regime the background dominates over the theoretical signal. Note that this latter area of parameter space provides the “maximum likelihood” (or χ^2 minimum); there is no neutrino signal and we calculate only limits on parameters. In fact, there is a slight deficit of counts with respect to the background calculated from the time before the supernova; otherwise we might expect to see a weak maximum likelihood somewhere in the large- $m_\nu\tau$ regime.

In Figure 3, we show a single contour of χ^2 in the $m_\nu\tau$ - B_γ plain. From a “Bayesian” viewpoint, this figure represents the complete inference from the data; we obviously would prefer to quote limits on the parameter space.

Because χ^2 is constant for both large and small values of the parameters $m_\nu\tau$, we must be careful in normalizing our probabilities. Without any other information to guide us, we might expect the appropriate prior to use would be either $p(\theta) \propto \text{const}$ or $p(\theta) \propto 1/\theta$ corresponding to constant probability per linear and logarithmic interval, respectively; however, neither of these choices converge when integrated out to infinity multiplied by a constant likelihood. These priors, when normalized, assign a vanishing weight to any finite region of parameter space; usually, χ^2 goes to infinity like the square of some parameter in a linear gaussian model with independent errors, so the wings are automatically disfavored regardless of the prior.

To get meaningful results in this case we must make sure that the interesting regime of $-6 \lesssim \log B_\gamma < 0$, $5 \lesssim \log(m_\nu\tau/\text{keV sec}) \lesssim 13$ is given finite prior weight. This corresponds to choosing a prior with some cutoff outside of this region, and will result in a limit corresponding to a value of χ^2 somewhere along the slope between its small and large values. To connect with other analyses, we will adopt the following “frequentist” procedure. We will assume that χ^2 is distributed in a χ^2 distribution considering the data as a frequentist random variable. We have 550 time bins and four detectors, so there are 2200 degrees of freedom. For this distribution, a one-sigma fluctuation corresponds to $\Delta\chi^2 = 2230$, a three-sigma (or 99%) fluctuation to $\Delta\chi^2 = 2357$; we choose the latter as our limit; from the shape of the likelihood function it is clear that any comparable $\Delta\chi^2$ will give similar bounds. We also note that a signal in the small- $m_\nu\tau$ regime may not be detectable with this algorithm; the absence of a local minimum in that region, however, implies that this should not be a significant worry. (We have also used a χ^2 defined rigorously from the true Poisson distribution, which effects the limits on the parameters by somewhat under one order of magnitude throughout.) The allowed region is shown in Figure 3. It corresponds to

$$B_\gamma < 3 \times 10^{-7} \quad m_\nu\tau \lesssim 10^6 \text{ keV sec} \quad (13)$$

$$B_\gamma < 2 \times 10^{-13} \frac{m_\nu\tau}{\text{keV sec}} \quad m_\nu\tau \gtrsim 10^6 \text{ keV sec.} \quad (14)$$

for neutrinos with a temperature of 8 MeV. (The limits scale roughly as T^{-2} .) The latter bound corresponds to $m_\nu\tau_\gamma > 5 \times 10^{12}$ keV sec. This is less restrictive than the limits of Oberauer et al. [4], due to the fact that the PVO GBD could only detect gamma rays with energies below 3 MeV, compared to 25 MeV for the SMM satellite. However, we believe this analysis to be more rigorous and the PVO data to be of higher quality.

IV. OTHER DECAY MODES

So far we have considered only photons produced from the simplest radiative decay of a massive neutrino species: $\nu \rightarrow \nu' \gamma$, but many other channels are possible. For these reactions, where the rest-frame photon energy is no longer given by the simple $\bar{k} = m_\nu/2$, we must allow for a distribution of decay products: $f(\bar{k}, \bar{\mu}) d\bar{k} d\bar{\mu}$ gives the fraction of photons produced with rest-frame energy \bar{k} into angle $\bar{\mu} = \cos \bar{\theta}$. Then, the final spectrum is given by

$$\frac{dN}{dkdt} = \frac{1}{4\pi D^2} \frac{B_\gamma}{\tau} \int dE L_\#(E) \int d\bar{\mu} f\left[\frac{k}{\gamma(1+v\bar{\mu})}, \bar{\mu}\right] e^{-\gamma(1+v\bar{\mu})t/\tau} \quad (15)$$

where we have still assumed that the decays occur near the supernova, and the integral is taken from k to ∞ . (We discuss the case of large masses, $m_\nu \gtrsim T_\nu$ in Sec. V below.) For the appropriate choice of $f(\bar{k}, \bar{\mu}) \propto \delta(\bar{k} - m_\nu/2) f_1(\bar{\mu})$, we reproduce the earlier 2-body decay formula, Eq. (7).

In particular, we consider the bremsstrahlung process, $\nu \rightarrow \nu' e^+ e^- \gamma$, where typically $\nu = \nu_\tau$, $\nu' = \nu_e$. Because this is no longer a two-body decay, the spectrum in angle and energy of the daughter photons is considerably more complicated, and there has been no exact calculation performed. Following Oberauer et al. [4], we make several simplifying assumptions. First, we assume isotropy of the photons in the rest frame of the neutrinos; this is reasonable if the helicity states of the parent neutrinos are produced in equal numbers. This gives $f(\bar{k}, \bar{\mu}) = f(\bar{k})/2$ (which still implicitly depends on $\bar{\mu}$ through the Lorentz transformation to the lab frame).

Up to factors of order unity, we assume after Oberauer et al. that the bremsstrahlung energy spectrum is given by

$$\frac{d\Gamma_{\text{br}}}{dk} \simeq \frac{\alpha}{\pi} \frac{\Gamma_0}{k} = \frac{\alpha}{\pi} \frac{1}{k \tau_e} \quad (16)$$

where Γ_0 and τ_e refer to the process without a daughter photon: $\nu \rightarrow \nu' e^+ e^-$, allowing us to absorb a branching ratio factor into $\tau_e = \tau/B_e$. Now, we can write the gamma-ray flux as

$$\begin{aligned} \frac{dN}{dkdt} &= \frac{1}{4\pi D^2} \frac{1}{m_\nu \tau_e} \frac{\alpha}{\pi} \int dE L_\#(E) \frac{E}{k} \int_{-1}^{+1} \frac{d\bar{\mu}}{2} (1+v\bar{\mu}) e^{-\gamma(1+v\bar{\mu})t/\tau} \\ &= \frac{1}{4\pi D^2} \frac{1}{m_\nu \tau_e} \frac{\alpha}{\pi} \int dE L_\#(E) \frac{E}{k} \frac{1}{2v} \left(\frac{m_\nu \tau}{Et} \right)^2 e^u (u-1) \Big|_{u=-(1-v)Et/m_\nu \tau}^{-(1+v)Et/m_\nu \tau} \end{aligned} \quad (17)$$

If we assume that $\gamma t \ll \tau$, the angular average becomes unity. (A Maxwell-Boltzmann distribution for $L_\#(E)$ then reproduces the expression of Eqs. (6-7) of Oberauer et al. [4]) With the additional assumption that a negligible fraction of the decays occur inside of the progenitor and that detected gamma rays have energies $k \ll T$ (i.e., $E_{\text{min}} \ll T$), we get the simple formula

$$\frac{dN}{dkdt} = \frac{E_T}{4\pi D^2} \frac{1}{m_\nu \tau_e} \frac{\alpha}{\pi} \frac{1}{k} \quad (18)$$

where E_T again gives the total energy in the decaying neutrino species. Note that this expression is correct for any form of $L_\#(E)$. At early times the flux is constant, although there will be an exponential decay for $\gamma t \gtrsim \tau$. In the two-body case, the timescale for gamma-ray detection is set by $m_\nu \tau / k$; in the bremsstrahlung case, by $m_\nu \tau / E \sim m_\nu \tau / T$.

Assuming that most neutrinos will have $E \sim T$, the assumptions we have made require that $t/\tau \ll m_\nu/T \ll 1$ for this formula to hold. Note that this is a restriction on the parameter space of m_ν and τ , since we have approximately 8000 sec of data, and the temperature is of order 8 MeV for any neutrino species.

Because the flux is proportional to $1/m_\nu \tau_e$, we will be able to put limits on the combination $m_\nu \tau_e = m_\nu \tau / B_e$. Moreover, because the dependence is the same as the large- $m_\nu \tau$ regime of the 2-body decay case (cf. Eq. (8)), the probability density will be of the same form. When the variance is dominated by the signal, $\chi^2 \sim \sum S_{ij} \propto 1/m_\nu \tau_e$; when it is dominated by the background, $\chi^2 \sim \text{const}$ (the same constant as in the 2-body case). The crossover occurs at $m_\nu \tau_e \approx 10^{15}$ keV sec. The value of χ^2 for this model is shown in Figure 4. Using the same quasi-frequentist definition of a 99% confidence level gives limits of

$$m_\nu \tau_e > 1.5 \times 10^{12} \text{ keV sec} \quad \text{or} \quad \frac{B_e}{m_\nu \tau} < 7 \times 10^{-13} \text{ keV}^{-1} \text{ sec}^{-1}. \quad (19)$$

Because the bremsstrahlung spectrum peaks at a lower energy, and due to the long time baseline of the PVO data, this limit is comparable to other SN1987A limits for this decay channel [2,4], and we believe more reliable, due to the higher-quality data set.

V. VERY MASSIVE NEUTRINOS

All of these expressions are considerable more complicated in the case $m \gtrsim T$. We will still assume a zero-chemical-potential FD distribution, this time applying to massive particles:

$$L_\#(E, m_\nu) = \frac{120}{7\pi^4} \frac{E_T}{T_\nu^4} j(m_\nu/T_\nu) \frac{E \sqrt{E^2 + m_\nu^2}}{1 + e^{E/T_\nu}}, \quad (20)$$

with a ‘‘suppression factor,’’ $j(x)$, along with the requirement that $E > m_\nu$. The factor $j(x)$ is just the usual Boltzmann suppression ($j(x) \propto x^{3/2} e^{-x}$, for $x \gg 1$); we use an approximation that is good for $m/T \lesssim \text{few}$, $j(x) \simeq \exp(-0.15x^2)$. (Sigl & Turner [7] have calculated the effect of the changing neutrinosphere temperature and radius on this naive expectation; the effect is small for $m_\nu \lesssim 40$ MeV, at least for $\tau \gtrsim 10^{-2}$ sec.) For low-mass neutrinos we assumed $k > m_\nu$; now, we can only integrate over neutrino energies greater than $\max(m_\nu, k)$. In this expression, $E_T \simeq 10^{53}$ erg remains the total energy for the low-mass case; the total energy released is $E_T j(m_\nu/T_\nu)$; which for large masses is less than 10^{53} erg since $j < 1$.

In addition to the mass threshold effects, with massive neutrinos ($m_\nu \gtrsim k$) we must now take into account the loss of any photons produced inside the envelope of the supernova, $R_{\text{env}} = 100c$ sec. Thus, we require that $vt_d > R_{\text{env}}$, or

$$E > E_{\text{env}} = m_\nu \sqrt{1 + \left(\frac{R_{\text{env}} m_\nu}{2kt} \right)}. \quad (21)$$

Note that $E_{\text{env}} > m_\nu$, so this supersedes the requirement that $E > m_\nu$, but the requirement that $E > k$ remains. Thus, we must integrate over neutrino energies from $E_{\text{min}} = \max(k, E_{\text{env}})$. This integral, the equivalent of $h(k/T)$ above, cannot be done in closed form, but again it can be approximated by a gaussian (at least for the isotropic case $f(E, k) = 1/p$):

$$\frac{dN}{dk dt} = \frac{B_\gamma}{4\pi D^2} j(m_\nu/T_\nu) \frac{120}{7\pi^4} \frac{E_T}{T_\nu^2} \frac{2k}{m_\nu \tau} e^{-2kt/m_\nu \tau} g(E_{\text{min}}/T) \quad (22)$$

with

$$g(x) = \int_x^\infty dy \frac{x}{1+e^y} \simeq \frac{\pi^2}{12} e^{-0.2x^2}; \quad (23)$$

the factor of 0.2 in the exponent approximates the shape of the integral for $x \lesssim \text{few}$. This differs from the massless case by a total suppression factor

$$\frac{j(m_\nu/T_\nu) g(E_{\text{min}}/T)}{h(k/T)} \simeq \exp \left[-0.15 \left(\frac{m_\nu}{T_\nu} \right)^2 - 0.2 \left(\frac{E_{\text{min}}}{T_\nu} \right)^2 \right] \quad (24)$$

Since $E_{\text{min}} \geq m_\nu$, this is always less than $\exp[-0.35(m_\nu/T_\nu)^2]$ for interesting masses $m_\nu \gtrsim T_\nu$; unfortunately, the time now appears in the expression for E_{min} , so the dt integral is no longer trivial. First, then, let us consider the suppression factor if we ignore the effect of decays inside the supernova envelope, integrating from $E_{\text{min}} = \max(m_\nu, k)$. Then the time integral can be done as in the low-mass case, and we can simply write down the time-independent suppression factor $s = j(m_\nu/T)g(E_{\text{min}}/T) \simeq j(m_\nu/T)g(m_\nu/T)$ (if we first set $h(k/T) = 1$ when numerically calculating the limits as above).

These mass effects enable us to break the degeneracy between m_ν and τ , at the price of requiring the more complicated analysis of a three-dimensional parameter space. To simplify matters, we will base the results for massive neutrinos directly on the limits from the low-mass case. That is, we will calculate the limits as before, and then apply the suppression factor at the end. We can do this because the suppression factor comes into the expression for the flux in exactly the same way as the branching ratio B_γ , so we translate limits on B_γ in the massless case to limits on $B_\gamma \times s$, where s is the k -independent part of the suppression factor. In addition, we do the calculation for an isotropic decay, and assuming $k < m_\nu$. For two-body decay, this results in the limit

$$sB_\gamma < 3 \times 10^{-7} \quad m_\nu \tau \lesssim 10^6 \text{ keV sec} \quad (25)$$

$$sB_\gamma < 6 \times 10^{-14} \frac{m_\nu \tau}{\text{keV sec}} \quad m_\nu \tau \gtrsim 10^6 \text{ keV sec.} \quad (26)$$

If we allow the effect of decays inside the progenitor envelope, the calculation is somewhat more complicated, and the integral over each time bin can no longer be done in closed form. We must now recompute everything at each pair of m_ν and τ . We show the results of such a calculation for several values of the neutrino mass in Figure 6; the limits are not too different from those with the simpler time-independent suppression.

For the bremsstrahlung process, the suppression of a high-mass neutrino flux is simpler to calculate because the required integral is simply $\int dE EL_\#(E)$, the total energy in the

massive neutrino species (cf. Eq. 18; we have again assumed $k \ll T$, so the initial integral over μ simplifies). Again, we integrate from the same $E_{\min} = \max(k, E_{\text{env}})$; the suppression is given by the Boltzmann factor $j(E_{\min}/T)$. For much of parameter space, this is simply the expected $j(m_\nu/T)$. Again, the time-dependence of E_{env} does not change the limits significantly. The allowed parameter space for the bremsstrahlung process for a mass of 30 MeV is

$$\frac{m_\nu \tau_e}{j(m_\nu/T)} > 1.5 \times 10^{12} \text{ keV sec} \quad \text{or} \quad j(m_\nu/T) \frac{B_e}{m_\nu \tau} < 7 \times 10^{-13} \text{ keV}^{-1} \text{ sec}^{-1}. \quad (27)$$

VI. NEARLY DEGENERATE NEUTRINOS

Thus far, we have assumed that the daughter neutrino in the $\nu \rightarrow \nu' \gamma$ channel is much less massive than the parent neutrino. If, however, the mass of the daughter is appreciable, the energy of the photon will be decreased by a factor $\delta m^2/m^2 \equiv (m_1^2 - m_2^2)/m_1^2$. This may improve our limits: It would shift the bulk of the photons down from energies too high to detect into one or more of the energy channels of the PVO detector. For massless daughter neutrinos, of order 1/10 of the photons can be detected; therefore we might expect limits as much as an order of magnitude stronger. In fact, for this case, the lower energy window of the PVO detector, down to 0.1 MeV, compared with the SMM window, sensitive only above 4.1 MeV, is actually an advantage.

To make the matter more precise, we see that in the case of nearly degenerate neutrinos, we make the change

$$f(E, k) dk \rightarrow f[E, (m^2/\delta m^2)k] (m^2/\delta m^2) dk \quad (28)$$

where we now are constrained to have photon energies $k < (\delta m^2/m)E$. This, in turn, results in changing $h(k/T) \rightarrow (m^2/\delta m^2)h[(m^2/\delta m^2)k/T] \approx (m^2/\delta m^2)h(k/T)$, up to a constant of order one. As expected, the flux is enhanced by a factor of $(m^2/\delta m^2)$ at most energies, so we can make the substitution $B_\gamma \rightarrow B_\gamma(m^2/\delta m^2)$ in our limits.

In particular, we are interested in two regimes of $\delta m^2/m^2$. To account for the recent evidence of a neutrino mass eigenstate, we set $m = 3 \text{ eV}$ [10]. The most attractive solution to the solar neutrino problem requires $\delta m^2 = 3 \times 10^{-5} \text{ eV}^2$, so $\delta m^2/m^2 = 3 \times 10^{-6}$ [11]. To account for the atmospheric ν_μ deficit, $\delta m^2 = 1 \times 10^{-2} \text{ eV}^2$, so $\delta m^2/m^2 = 1 \times 10^{-3}$ [12]. Roughly, we see that our limits will become stronger by factors order 3×10^5 and 10^3 , respectively. In Figure 5, we show the limits on the neutrino parameters for these two cases.

VII. LONG LIFETIMES

For long lifetimes (such that the average decay time of the neutrino is comparable to or longer than the travel time to the detector), the above formalism becomes too cumbersome, because we must integrate over a complicated set of possible paths for the neutrino and daughter photon. In this case, we will make several simplifications. At first, we will only concern ourselves with the total gamma-ray fluence from the decays, integrated over time. Then, we will integrate over the decay time, $0 \leq t_d \leq D$:

$$\frac{dN}{dk} = \frac{B_\gamma}{4\pi D^2} \int dE L_\#(E) f(E, k) [1 - e^{-Dm_\nu/E\tau}] \quad (29)$$

where, as before, $f(E, k)$ gives the fraction of neutrinos with energy E decaying into photons with energy k . This expression is to be compared with those presented in [2]. The cost of the simplicity of this expression is the inability to determine the exact time of a photon's arrival. For neutrinos and photons travelling on a straight path ($\mu = 1$, appropriate for relativistic particles), the arrival time after the supernova light-pulse is $t = t_d(1 - v) \simeq (t_d/2)m_\nu^2/E^2$. For long lifetimes, we will be concerned with neutrinos that decay late in their flight: $t_d \sim D$. Using this as a typical e -folding time, we have the ansatz that $(dN/dt) \propto \exp(-2tE^2/Dm_\nu^2)$. For $t \gtrsim (D/2)m_\nu^2/E^2$, this should express the character of the time-dependence. Two effects are explicitly missing from this formula: the extra time delay from non-straight paths (of the same order as the delay already considered) and the photon energy dependence of the time-delay. Moreover, the time-dependence will not have exactly this shape; for short times it does not contain the expected slow rise from zero flux, so it is probably safest to use this formula integrated over the entire duration of the experiment, and not rely on the detailed time evolution, leaving us finally with

$$\left. \frac{dN}{dk} \right|_{\delta t} \simeq \frac{B_\gamma}{4\pi D^2} \int dE L_\#(E) f(E, k) (1 - e^{-2\delta t E^2/Dm_\nu^2}) (1 - e^{-Dm_\nu/E\tau}). \quad (30)$$

Unfortunately, the integration over neutrino energy E is considerably more complicated than before, but we can approximate the two exponential decays for various regimes:

$$(1 - e^{-2\delta t E^2/Dm_\nu^2}) (1 - e^{-Dm_\nu/E\tau}) \approx \begin{cases} 1; & m \lesssim E\sqrt{2\delta t/D}, \quad m_\nu/\tau \gtrsim E/D \\ 2\delta t E^2/Dm_\nu^2; & m \gtrsim E\sqrt{2\delta t/D}, \quad m_\nu/\tau \gtrsim E/D \\ Dm_\nu/E\tau; & m \lesssim E\sqrt{2\delta t/D}, \quad m_\nu/\tau \lesssim E/D \\ 2\delta t E/m_\nu\tau; & m \gtrsim E\sqrt{2\delta t/D}, \quad m_\nu/\tau \lesssim E/D. \end{cases} \quad (31)$$

Numerically, these breaks occur at

$$m \simeq E\sqrt{2\delta t/D} \simeq 680 \text{ eV} \frac{E}{12 \text{ MeV}} \left(\frac{\delta t}{8500 \text{ sec}} \right)^{1/2}; \quad m_\nu/\tau \simeq E/D \simeq 2 \times 10^{-6} \text{ eV/sec} \frac{E}{12 \text{ MeV}}. \quad (32)$$

where $E = 12 \text{ MeV}$ is a typical energy for a $T_\nu = 8 \text{ MeV}$ blackbody. In terms of the lifetime τ , the latter limit occurs at $\tau \simeq Dm_\nu/E \approx 5 \times 10^5 \text{ sec}$ (m_ν/eV)—for masses $m_\nu \sim 1 \text{ MeV}$, this is roughly $\tau \sim D$. When this is proportional to δt , the flux is approximately constant; otherwise, the entire pulse is detected (and its shape is irrelevant). Putting all of this together, and doing the integral over E gives

$$\left. \frac{dN}{dk} \right|_{\delta t} \approx \frac{B_\gamma}{4\pi D^2} \frac{120}{7\pi^4} \times \begin{cases} \frac{E_T}{T_\nu^2} h_0(k/T) \\ 2 \frac{E_T}{m_\nu^2} \frac{\delta t}{D} h_2(k/T) \\ \frac{E_T m_\nu}{T_\nu^3} \frac{D}{\tau} h_{-1}(k/T) \\ 2 \frac{E_T}{T_\nu m_\nu} \frac{\delta t}{\tau} h_1(k/T) \end{cases} \quad (33)$$

where $h_n(y) = \int_y^\infty x^{n+1}/(1 + \exp x)$ is similar to $h(y)$ above, and the four cases correspond to those in Eq. (31). Here, we have assumed an isotropic distribution of decays in the rest frame. As before, these expressions hold for $m_\nu \lesssim T_\nu$ and must be modified with the appropriate suppression factor otherwise.

Now, we can just put these results through our statistical machinery and find limits on the parameters. We will write the flux as

$$\left. \frac{dN}{dk} \right|_{\delta t} \approx \frac{1}{4\pi D^2} \frac{120}{7\pi^4} \frac{E_T}{T_\nu^2} h(k/T) \times B_\gamma A(m_\nu, \tau) \quad (34)$$

where A is the appropriate dimensionless combination of m_ν and τ , along with D , T_ν and δt ; the data give us limits on A in each (m_ν, τ) regime (assuming that we can write our prior information as a simple probability distribution for A). This gives an approximate 99% confidence limit of $B_\gamma A \lesssim 1 \times 10^{-6}$ for $T_\nu = 8$ MeV or,

$$\begin{aligned} B_\gamma &\lesssim 1 \times 10^{-6} \left(\frac{T_\nu}{8 \text{ MeV}} \right) & m_\nu &\lesssim 0.4 \text{ keV}, m_\nu/\tau \gtrsim 1.2 \times 10^{-9} \text{ keV/sec}; \\ m_\nu &\gtrsim 155 \text{ keV} B_\gamma^{1/2} & m_\nu &\gtrsim 0.4 \text{ keV}, m_\nu/\tau \lesssim 1.2 \times 10^{-9} \text{ keV/sec}; \\ \frac{B_\gamma m_\nu}{\tau} &\lesssim 1.4 \times 10^{-15} \text{ keV sec}^{-1} \left(\frac{T_\nu}{8 \text{ MeV}} \right)^3 & m_\nu &\lesssim 0.4 \text{ keV}, m_\nu/\tau \gtrsim 1.2 \times 10^{-9} \text{ keV/sec}; \\ m_\nu \tau &\gtrsim 1.4 \times 10^{14} \text{ keV sec} B_\gamma \left(\frac{T_\nu}{8 \text{ MeV}} \right)^{-1} & m_\nu &\gtrsim 0.4 \text{ keV}, m_\nu/\tau \lesssim 1.2 \times 10^{-9} \text{ keV/sec}. \end{aligned} \quad (35)$$

Where the regimes overlap, these limits are comparable to those calculated with the more detailed models above—because we can only calculate limits on parameters, the details of the data and the analysis are unimportant (in fact, the limits of Eq. (35) are *stronger* than, for example, Eq. (13) above; the earlier, more detailed calculation is probably the more appropriate limit). Again, for neutrinos with $m_\nu \gtrsim T_\nu$, these limits are modified with $B_\gamma \rightarrow s B_\gamma$.

For the bremsstrahlung channel, the flux is changed due to the different kinematics of the decay (i.e., the rest frame spectrum of Eq. (16)):

$$\left. \frac{dN}{dk} \right|_{\text{brem}} = \frac{\alpha}{\pi} \frac{2T_\nu^2}{km_\nu} \times \left. \frac{dN}{dk} \right|_{\text{2-body}} \quad (36)$$

(in addition, the functions h_n should also be modified to h_{n+2}). This is a significant increase in flux at for $km_\nu \lesssim T^2$. As before, we see that the bremsstrahlung spectrum at the detector is proportional to $1/k$. Now, the limits correspond to $B_\gamma AT/m \lesssim 3 \times 10^{-5}$ or

$$\begin{aligned}
m_\nu &\gtrsim 2.7 \times 10^8 \text{ keV} B_\gamma \left(\frac{T_\nu}{8 \text{ MeV}} \right) & m_\nu &\lesssim 0.4 \text{ keV}, m_\nu/\tau \gtrsim 1.2 \times 10^{-9} \text{ keV/sec}; \\
m_\nu &\gtrsim 800 \text{ keV} B_\gamma^{1/3} \left(\frac{T_\nu}{8 \text{ MeV}} \right) & m_\nu &\gtrsim 0.4 \text{ keV}, m_\nu/\tau \lesssim 1.2 \times 10^{-9} \text{ keV/sec}; \\
\tau/B_\gamma = \tau_e &\gtrsim 1.9 \times 10^{16} \text{ sec} & m_\nu &\lesssim 0.4 \text{ keV}, m_\nu/\tau \gtrsim 1.2 \times 10^{-9} \text{ keV/sec}; \\
m_\nu &\gtrsim 1.4 \times 10^8 \text{ keV} \left(\frac{\tau/B_\gamma}{\text{sec}} \right)^{-1/2} \left(\frac{T_\nu}{8 \text{ MeV}} \right) & m_\nu &\gtrsim 0.4 \text{ keV}, m_\nu/\tau \lesssim 1.2 \times 10^{-9} \text{ keV/sec}. \quad (37)
\end{aligned}$$

VIII. DISCUSSION

SN1987A not only confirmed astrophysicists' standard model of the Type II (core collapse) supernovae, but also provided a laboratory for studying the properties of neutrinos. The fluence of neutrinos at earth was approximately 10^{11} cm^{-2} per species. This large fluence and the space-borne gamma-ray detectors operating on SMM and PVO have allowed stringent limits to be placed on the radiative decay of neutrinos.

Although there is only 232 seconds of data, in a single time bin, the SMM detectors were sensitive up to energies of 25 MeV, and so would have more likely to detect gamma rays from decaying SN1987A neutrinos with a temperature of 4-8 MeV. Because the limits are determined by the region of parameter where the background becomes comparable to the signal (see the discussion in Sec. III above), the long time base and greater resolution is actually of little use in determining the limits on the parameters. To show this, we have performed our analysis with the SMM data as presented in [4], using crude estimates of the response matrix and effective area of the detector. As expected, the results are comparable to, although slightly weaker than, those quoted in that reference. However, in the case where the gamma rays are produced by bremsstrahlung or the neutrino mass states are nearly degenerate, the PVO limits are comparable or more stringent. Finally, the amount of and quality of the PVO data adds additional confidence to the SMM-based limits.

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REFERENCES

- [1] J. Ellis and Schramm, D.N., Phys. Rev. **D45**, VI.42 (1992) and references therein.
- [2] E. L. Chupp *et al.* Phys. Rev. **D62**, 505 (1989); E. W. Kolb and M. S. Turner, Phys. Rev. **D62**, 509 (1989).
- [3] A. H. Jaffe, E. Fenimore, and M. S. Turner, in S. Godfrey & P. Kalyniak, *Beyond the Standard Model III*, World Scientific (1993); A. H. Jaffe in C.H. Albright *et al*, *The Fermilab Meeting: DPF '92*, World Scientific (1993).
- [4] L. Oberauer *et al.*, Astropart. Phys. **1**, 377 (1993).
- [5] S. Dodleson, J.A. Frieman and M.S. Turner, Phys. Rev. Lett. **68**, 2572 (1992); J.M. Soares and L. Wolfenstein, Phys. Rev. **D40**, 3666 (1989); A. Burrows, D. Klein and R. Gandhi, Phys. Rev. **D45**, 3361 (1992).
- [6] R.W. Klebsadel, W.D. Evans, J.P. Glore, R.E. Spalding and F.J. Wymer, *IEEE Trans. Geosci. Rem. Sens.* **GE-18**, 76 (1980).
- [7] G. Sigl and M. S. Turner, Phys. Rev. **D51**, 1499, (1995).
- [8] A. Burrows, Ann. Rev. Nucl. Part. Sci., **40**, 181 (1990) and references therein.
- [9] S. Falk and Schramm, D.N., Phys. Lett. **B79**, 511 (1978).
- [10] C.A. Thanasoupoulos *et al*, Phys. Rev. Lett. **75**, 2650 (1995); J.E. Hull, *ibid* **75**, 2654 (1995).
- [11] See, e.g., S. Parke, Phys. Rev. Lett. **74**, 839 (1995)
- [12] K.S. Hirata *et al*, Phys. Lett. **B 280**, 146 (1992); Y. Fukuda *et al*, *ibid*, **335**, 237 (1994); R. Becker-Szendy *et al*, Phys. Rev. **D 46**, 4720 (1992).

FIGURES

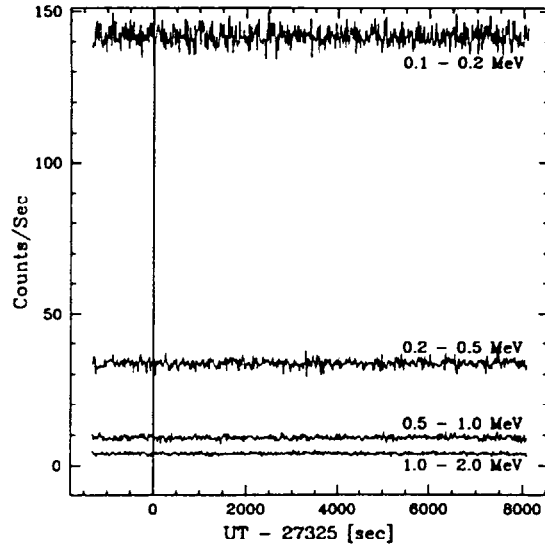


FIG. 1. The PVO GBD data for the time immediately before and after the arrival of the light from SN1987A at the PVO spacecraft ($UT = 27325$). Time bins are either 12 or 16 sec; we show the average counts per second in each bin, for each energy channel, as marked.

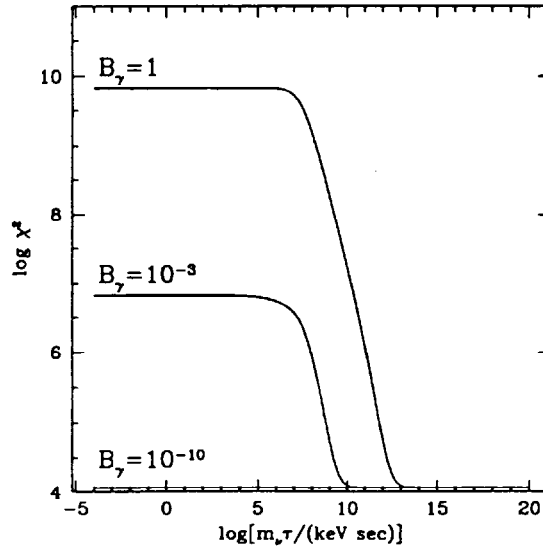


FIG. 2. The value of the χ^2 statistic, defined in the text, as a function of the parameter $m_\nu \tau$, for values of the branching ratio B_γ as marked, for the decay process $\nu \rightarrow \nu' \gamma$.

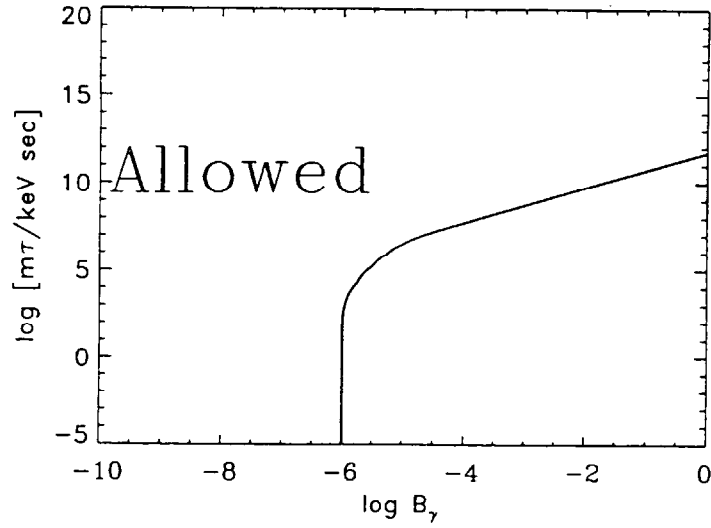


FIG. 3. Allowed region of the $m_\nu \tau$ - B_γ plane, for the 2-body decay process $\nu \rightarrow \nu' \gamma$, corresponding to $\Delta\chi^2 \leq 2357$ (see text). Here and below, contours continue to infinity as long as the appropriate assumptions, discussed in the text, still hold.

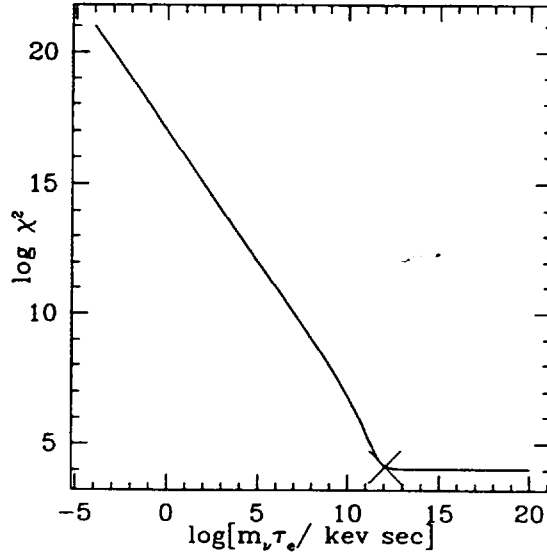


FIG. 4. The value of the χ^2 statistic, defined in the text, as a function of the parameter $m_\nu \tau_e = m_\nu \tau / B_e$, for the bremsstrahlung process. Also shown is the location of the $\Delta\chi^2 \leq 2357$ limit.

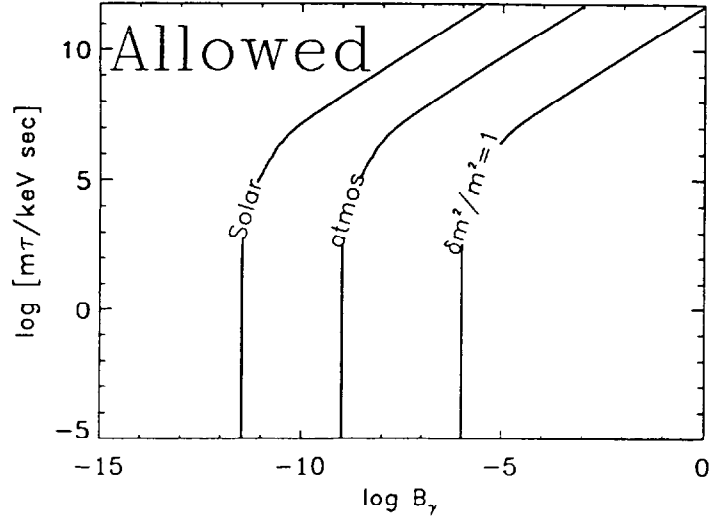


FIG. 5. Allowed region of the $m_\nu \tau - B_\gamma$ plane, for nearly degenerate neutrinos, with $\delta m^2/m^2 = 10^{-3}$ (“atmospheric”), 3×10^{-6} (“solar”), as well as $\delta m^2/m^2 = 1$, as labelled.

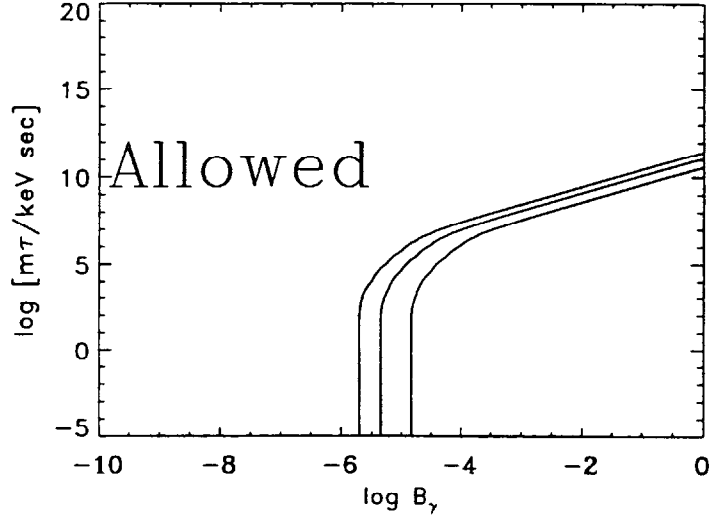


FIG. 6. Allowed region of the $m_\nu \tau - B_\gamma$ plane, for neutrinos of mass 20 MeV, 30 MeV, and 40 MeV (from left to right), for the 2-body decay process, corresponding to $\Delta\chi^2 \leq 2357$ (see text).